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Non-intercommuting configurations in the collisions of type I $U(1)$ cosmic strings

L. M. A. Bettencourt* and T. W. B. Kibble

The Blackett Laboratory, Imperial College, London SW7 2BZ, U.K.

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Abstract

It is shown that for small relative angle and kinetic energy two type I $U(1)$ strings can form bound states upon collision instead of the more familiar intercommuting configuration. The velocity below which this may happen is estimated as function of the ratio of the coupling constants in the theory, crossing angle and initial kinetic energy.

*e-mail: l.bettencourt@ic.ac.uk

Cosmic Strings formed at a grand unified symmetry breaking phase transition [1] are a strong candidate to solve one of the most outstanding problems of cosmology namely that of structure seeding [2]. The standard cosmic string scenario assumes that strings after the phase transition rapidly approach the limit of zero width and become henceforth well described in terms of the Nambu-Goto action. The other necessary ingredient to specify the evolution of the network is that at each collision strings should exchange ends or in other words should intercommute. In particular, whenever two segments of the same long string intersect a loop will be formed. Loops of string, in turn, are not globally topologically stable and will shrink by emitting their energy presumably as gravitational radiation. This process allows for a faster decrease of the energy density in strings and precludes the possibility of a string dominated universe.

This whole construction rests crucially on the assumption that the outcome of a string collision always corresponds to intercommuting. However, strings are in rigour classical non-trivial solutions of a variety of field theories displaying spontaneous symmetry breaking [1]. The preferred model for realising string dynamics, however, is the abelian Higgs model

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}|(\partial_\mu + iqA_\mu)\phi|^2 - \frac{\lambda}{8}(|\phi|^2 - \eta^2)^2. \tag{1}$$

In what follows we will also confine ourselves to it and its $U(1)$ string solutions [3].

The only way to establish intercommuting as the necessary outcome of the collision between two cosmic strings is to study the dynamics of the full field theory solutions. This, however, involves solving a set of non-linear equations for the fields which proves to be a rather difficult task in most circumstances. One way out is to perform simulations of collisions by solving the theory numerically. This was done for several values of the ratio of the coupling constants in (1), $\beta = \sqrt{\frac{q^2}{\lambda}}$, supporting the conjecture that global, type II and critically coupled strings (i.e., $\beta \leq 1$) intercommute under essentially all circumstances [4–6]. Similar studies regarding the behaviour of type I strings are much scarcer and seem to open the possibility for the existence of more complicated configurations [7].

For a collision between two cosmic strings to be analytically understood in terms of its

underlying field theory one needs to look, in the first instance, at vortex dynamics. Indeed, both strings, at the collision point, can be projected onto a set of two orthogonal planes, thus reducing the problem to the study of vortex dynamics in two dimensions[1] [5,9]. In this picture intercommuting results from the annihilation of a vortex-antivortex pair in one of these planes and the scattering at right angles of the vortex-vortex pair on the other. The scattering at right angles, in turn, is only fully understood analytically in the critically coupled regime [10]. This important result may, although possibly under more severe restrictions, generalise to non-critically coupled vortices. As a practical result the interactions should vanish at zero separation as is observed numerically [11], at least for β not too far from unity. This conjecture also seems to be well supported by several other numerical and semi-analytical recent studies of non-critically coupled vortex scattering dynamics [12,13]. It then seems likely that the effect of the interactions between cosmic strings should be felt at intermediate distances, i.e., of the order of several scalar field coherence lengths. The character of these is better known than their short distance counterpart. In particular, for $\beta > 1$ (type I) [8,11] these interactions are always attractive, regardless of the relative orientation of a pair of strings. In principle this leads to the possibility of formation of a bound state (characterised by the sum of the individual winding numbers of the two original strings) as the outcome of a sufficiently favourable collision between two string segments.

In this letter we explore this scenario by investigating what configurations could compete with the usual intercommuting as the outcome of the collision of two straight type I strings at an angle and under which circumstances these can be expected to prevail.

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[1]This construction should work particularly well for type II strings, when the character of the interactions is dominated by the gauge field and consequently is sensitive to spatial orientation [8]. The approximation of the dynamics of the whole string to that of vortices, however, excludes the possible excitation of modes along the string which can play an important role in dissipative effects.

A typical intercommuting event that occurs at the origin at $t = 0$ may be taken to be between two strings whose initial configuration is given (for $t < 0$) by

$$\begin{aligned} x_1 &= (t, \xi\sigma, -\eta\sigma, \zeta t) \\ x_2 &= (t, \xi\sigma, \eta\sigma, -\zeta t), \end{aligned} \tag{2}$$

where $\xi^2 + \eta^2 + \zeta^2 = 1$. Note that ζ is the velocity of approach. We can write $x_j = \frac{1}{2}\left[a_j(u) + b_j(v)\right]$, where $u = t + \sigma$ and $v = t - \sigma$.

Then,

$$\begin{aligned} a_1 &= u(1, \xi, -\eta, \zeta) \\ b_1 &= v(1, -\xi, \eta, \zeta), \end{aligned} \tag{3}$$

and

$$\begin{aligned} a_2 &= u(1, \xi, \eta, -\zeta) \\ b_2 &= v(1, -\xi, -\eta, -\zeta). \end{aligned} \tag{4}$$

If we have the normal type of intercommuting event, the solution for $t > 0$ is easy to describe. For $|\sigma| > t$, the previous solutions are still valid, but in the region where $u > 0$ and $v > 0$ the partners have swapped over. The solutions are as illustrated in figure 1. The "horizontal" pieces of string, in fig. 1, are described by

$$\frac{1}{2}\left[a_2(u) + b_1(v)\right] = (t, \xi\sigma, \eta t, -\zeta\sigma), \tag{5}$$

and

$$\frac{1}{2}\left[a_1(u) + b_2(v)\right] = (t, \xi\sigma, -\eta t, \zeta\sigma). \tag{6}$$

Note that the velocity on these sections is in the y-direction, not in the z-direction.

Can we then find an alternative solution, based on the assumption that a segment of $n = 2$ string is created at the moment of intersection? From symmetry it is easy to see that if such a segment is created it should be static and directed along the x-axis. Let us write

$\epsilon = \frac{2\mu}{\mu_2}$, where μ is the string tension for unit winding number, and μ_2 the corresponding value for $n = 2$. ϵ is naturally a function of β and, in particular for type I strings, we have $\epsilon(\beta > 1) > 1$. Then the solution for the $n = 2$ segment should be of the form

$$x_D = (t, \epsilon\sigma, 0, 0). \tag{7}$$

It is reasonable to assume that the ends of this segment must move with definite velocity. Let us take x_D to be the solution for σ in the range $-\kappa t < \sigma < \kappa t$, for some κ. Then the ends will move with speed $\kappa\epsilon$ (obviously $\kappa\epsilon < 1$). The overall shape must then be as illustrated in fig. 2, with four links at the same places as before receding with speed 1, joined to the doubled section by new straight segments. We can then write down the expressions for the string position on each of the straight segments, since we know the coordinates of the end-points

$$\begin{aligned}
x_3 &= \frac{1}{1-\kappa}\left((1-\kappa)t, -\epsilon\kappa(t+\sigma) + (\sigma+\kappa t)\xi, -(\sigma+\kappa t)\eta, -(\sigma+\kappa t)\zeta\right) \\
x_4 &= \frac{1}{1-\kappa}\left((1-\kappa)t, -\epsilon\kappa(t+\sigma) + (\sigma+\kappa t)\xi, (\sigma+\kappa t)\eta, (\sigma+\kappa t)\zeta\right) \\
x_5 &= \frac{1}{1-\kappa}\left((1-\kappa)t, \epsilon\kappa(t-\sigma) + (\sigma-\kappa t)\xi, -(\sigma-\kappa t)\eta, (\sigma-\kappa t)\zeta\right) \\
x_6 &= \frac{1}{1-\kappa}\left((1-\kappa)t, \epsilon\kappa(t-\sigma) + (\sigma-\kappa t)\xi, (\sigma-\kappa t)\eta, -(\sigma-\kappa t)\zeta\right).
\end{aligned} \tag{8}$$

Equivalently we can write,

$$\begin{aligned}
x_3 &= \frac{1}{2}\left(a_3 + b_1\right), & a_3 &= \frac{u}{1-\kappa}\left(1-\kappa, -2\epsilon\kappa + (1+\kappa)\xi, -(1+\kappa)\eta, -(1+\kappa)\zeta\right) \\
x_4 &= \frac{1}{2}\left(a_4 + b_2\right), & a_4 &= \frac{u}{1-\kappa}\left(1-\kappa, -2\epsilon\kappa + (1+\kappa)\xi, (1+\kappa)\eta, (1+\kappa)\zeta\right) \\
x_5 &= \frac{1}{2}\left(a_1 + b_5\right), & b_5 &= \frac{v}{1-\kappa}\left(1-\kappa, 2\epsilon\kappa - (1+\kappa)\xi, (1+\kappa)\eta, -(1+\kappa)\zeta\right) \\
x_6 &= \frac{1}{2}\left(a_2 + b_6\right), & b_6 &= \frac{v}{1-\kappa}\left(1-\kappa, 2\epsilon\kappa - (1+\kappa)\xi, -(1+\kappa)\eta, (1+\kappa)\zeta\right).
\end{aligned} \tag{9}$$

To satisfy the consistency conditions, we require

$$[2\epsilon\kappa - (1+\kappa)\xi]^2 + (1+\kappa)^2\eta^2 + (1+\kappa)^2\zeta^2 = (1-\kappa)^2, \tag{10}$$

or equivalently

$$4\epsilon^2\kappa^2 - 4\epsilon\kappa(1+\kappa)\xi + (1+\kappa)^2 = (1-\kappa)^2, \tag{11}$$

which yields

$$\epsilon^2\kappa - \epsilon(1+\kappa)\xi + 1 = 0. \tag{12}$$

Thus, the value of κ is given by

$$\epsilon\kappa = \frac{\epsilon\xi - 1}{\epsilon - \xi}. \tag{13}$$

Clearly a solution exists, if and only if,

$$\xi > \frac{1}{\epsilon}. \tag{14}$$

This is equivalent to requiring the angle between the strings, α, to be small, or the velocity of approach, ζ, to be small or both. We have $\xi^2 + \eta^2 = 1 - \zeta^2 = \frac{1}{\gamma^2}$ and consequently, (see fig. 1),

$$\xi = \frac{\cos(\frac{\alpha}{2})}{\gamma} > \frac{1}{\epsilon}. \tag{15}$$

The limit on the angle is then $\cos(\frac{\alpha}{2}) > \frac{\gamma}{\epsilon}$. It is not surprising that a "zipper" can exist only for small angles and/or velocities. In particular, for $\beta = 2$, $\epsilon^{-1} = 0.921$ [7], and the constraint (15) becomes quite severe.

Having shown that the formation of "zippers" is possible in the Nambu-Goto approximation we investigate how the consideration of the interactions between the two colliding strings alters the corresponding constraints on the collision parameters (15). In the Nambu-Goto approximation we have seen that the difference between the tension of the $n = 2$ string when compared to that of the two $n = 1$ strings added together, allowed for the formation of the "zipper". In this approximation such a difference is restricted to the $n = 2$ bridge only, disregarding the interactions between the two $n = 1$ "legs" at each end of it. By including these interactions we should then be able not only to render our former construction more realistic but also to relax the rather tight constraints it imposed.

Down to scalar coherence length separations it was shown [8] that the interaction energy between two $n = 1$ string elements at an angle α, is given approximately by

$$E_{\rm int}(d) = 2\pi\eta^2 \left[\cos(\alpha) K_0(m_A d) - K_0(m_S d)\right], \tag{16}$$

where $m_A = \eta q$ and $m_S = \eta\sqrt{\lambda}$.

On the other hand each vortex possesses a self-energy μ, which naturally emerges as the string tension in the Nambu-Goto approximation and can be seen as its inertial mass for the study of collisions. For general couplings μ has to be computed numerically. A best fit to its dependence on β yields [14]

$$\mu = 1.04\pi \frac{\eta^2}{(2\beta^2)^{0.195}}, \tag{17}$$

which should be valid to about 5% accuracy, in the range

$$0.01 < 2\beta^2 < 100.$$

We can then investigate the existence of bound states by matching the kinetic energy of two segments emerging from the collision, in the center-of-mass frame, to the potential energy of the system at a given separation. In so doing we are assuming that only a finite length of string contributes to the dynamics, at the collision point. We express this in terms of a simple cutoff, $l_{\rm eff}$, a length along the string measured from the point of collision and take the velocity of each string as seen from the center-of-mass frame, $v_{\rm CM}$, to be constant within such an interval[2]. This procedure yields

$$\int_0^{l_{\rm eff}} dz 2\mu(\gamma_{\rm CM} - 1) = 2\pi\eta^2 \int_0^{l_{\rm eff}} dz \left[K_0(m_S d(z)) - \cos(\alpha) K_0(m_A d(z))\right], \tag{18}$$

[2]We should note that for points in the vicinity of the collision point $v_{\rm CM}$, should be largest and then decreasing to zero smoothly with increasing distance along the string. Our procedure of introducing a simple cutoff $l_{\rm eff}$, gives us, therefore, a measure of the total kinetic energy in the direction of separation, but not of its distribution along the string.

which, in turn, leads to

$$v_{\rm CM} = \sqrt{\frac{a^2+2a}{(1+a)^2}}, \tag{19}$$

where

$$a = \frac{1}{1.04 l_{\rm eff}} (2\beta^2)^{0.195} \int_0^{l_{\rm eff}} dz \left[K_0(m_S d(z)) - \cos(\alpha) K_0(m_A d(z)) \right], \tag{20}$$

and

$$d(z) = d_0 + z \tan(\alpha). \tag{21}$$

We see, in particular, that if we take $l_{\rm eff}$ to infinity the integral in (20) should rapidly converge to a constant while the inverse proportionality of the velocity relative to $l_{\rm eff}$ ensures that the system will always be free. Conversely, in the limit of vanishing $l_{\rm eff}$, the picture for the collision of two vortices emerges, with the consequent drop in dependence on $l_{\rm eff}$. For $\alpha = 0$ the integrand in (20) becomes a constant and this results trivially. Physically, $l_{\rm eff}$ should depend on the parameters of the collision, namely on the angle of approach α and on the time-scale for the exchange of ends between the two strings. Determining $l_{\rm eff}$ would then require the detailed knowledge of the dynamics of the collision, which is unknown. In what follows we will, therefore, leave it as a free parameter and study the effect of its variation upon $v_{\rm CM}$.

The distance d_0 corresponds to the separation at which the kinetic energy should match the potential energy exactly. Because we expect expression (16) to break down at distances smaller than the coherence length of the scalar field [8] we take it to be $d_0 = 2m_S^{-1}$. This condition allows the system to scatter before the two segments are brought back together, under the effect of the interactions. It should also correspond to a value close to the maximum of the interaction energy, since, for smaller separations, it was shown numerically [11] that it should remain approximately constant as the separation vanishes, at least for β not too far from unity.

Figure 3 shows a plot of the dependence of the binding velocity, $v_{\rm CM}$, on β for four values of α and $l_{\rm eff} = 20m_S^{-1}$. We see the strong dependence on the angle of approach α.

In particular for orthogonal strings we expect the contribution from the potential energy to almost vanish and the system will be free. For intermediate angles, however, the binding velocity, $v_{\rm CM}$ can be naturally in the range of $0.1 - 0.2c$. The actual value depends quite sensitively on $l_{\rm eff}$. Figure 4 a), b), c) show the dependence of the binding velocity on β and $l_{\rm eff}$ for $\alpha = \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$, respectively. The dependence of the velocity on the angle of approach is shown in figure 5, for $\beta = 2$ and five values of $l_{\rm eff} = 5, 20, 50, 100, 200 m_S^{-1}$. In particular it becomes clear that the velocities only converge to a negligible value for α close to $\frac{\pi}{2}$. For intermediate angles the variation is relatively small allowing for a large range of parameters for which the velocity is relativistic.

Ultimately, one wants to relate the binding velocity (19) with the approach velocity of two typical strings in the network. In particular the collision needs to be reasonably inelastic, since otherwise the system could be trivially time-reversed, resulting in configurations with the strings at the same separation and speed before and after the collision. One way of parameterising this is by the use of an efficiency parameter, ρ, in the following way. Just before the two zeros of the scalar field superimpose, for very small separation, we have argued that the system's energy should be essentially kinetic. Then the same follows for the configuration just after the collision. However, the incoming energy will be channeled to all possible modes of an $n = 2$ vortex, among which is the one corresponding to the separation of the two $n = 1$ vortices. We then take ρ to be the ratio between the initial and final (kinetic) energies, in the separation mode, before and after the collision. Consequently, $0 \leq \rho \leq 1$. This, in turn, results in a relation between the initial and final velocities, before and after the collision

$$v_f^2 = \frac{\rho v_i^2}{1 - v_i^2(1-\rho)}, \tag{22}$$

where, v_f will be taken to be the above $v_{\rm CM}$. Once thus computed, v_i can be related to a velocity at a given separation by taking the effect of the interactions into account, for a given spatial configuration. The typical velocity quoted for a network of cosmic strings [15,16] is about $0.15c$.

Our estimates depend quite sensitively on two undetermined parameters $l_{\rm eff}$ and ρ. Their values can, in principle, be found by looking at simulations or by solving the short distance dynamics of strings more exactly analytically. If $l_{\rm eff}$ and ρ then turn out to be large, only a very small fraction of all collisions will have a "zipper" as the outcome. Subsequent collisions of these configurations with other strings may lead to the peeling or "unzipping" of these higher winding number bridges as was shown in simulations by Laguna and Matzner [7]. As such the difference between the evolution of such a network and those evolving under the effect of intercommuting alone, should be negligible. For sufficiently low values of $l_{\rm eff}$ and ρ, however, we have shown that a large fraction of all collisions of strings in the network would lead to local higher winding number bridges that could grow as "zippers". The overall evolution of such a network can then be considerably different. In principle loop formation under these circumstances could still occur, at the same rate, but such loops would remain connected to their mother string. This should have an effect on their subsequent evolution.

A network of type I strings can then be richer in string configurations than one which evolves simply by intercommuting at every collision. This should have consequences for the evolution of the characteristic lengths in a string network, which, in turn, could reflect in the way strings seed energy distribution anisotropies, hopefully with potentially observable consequences both in structure in the universe and anisotropies in the Cosmic Microwave Background Radiation.

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REFERENCES

[1] T. W. B. Kibble, J. Phys. A 9, (1976) 1387.

[2] Ya. B. Zeldovich, Mon. Not. Astron. Soc. 192, (1980) 663; A. Vilenkin, Phys. Rev. Lett. **46**, (1981) 1169; 46, (1981) 1496(E).

[3] H. B. Nielsen and P. Oleson, Nucl. Phys. B 61, (1973) 45.

[4] E. P. S. Shellard, Nucl. Phys B 283, (1987) 624.

[5] E. P. S. Shellard, in: Cosmic Strings: The current status, eds. F. S. Accetta and L. M. Kraus (World Scientific, 1988) p. 25.

[6] R. A. Matzner, Comput. Phys. 2 , (1989) 51.

[7] P. Laguna and R. A. Matzner, Phys. Rev. Letters 62 , (1989) 1948.

[8] L. M. A. Bettencourt and R. J. Rivers, Imperial College Preprint No. Imperial/TP/93-94/33.

[9] E. Copeland and N. Turok, unpublished.

[10] P. J. Ruback, Nucl. Phys B 296, (1988) 669.

[11] L. Jacobs and C. Rebbi, Phys. Rev. B 19, (1979) 4486.

[12] E. Meyers, C. Rebbi and R. Strilka, Phys. Rev. D 45, (1992) 1355.

[13] P. A. Shah, Cambridge Pre-print DAMTP 94-8

[14] H. M. Hodges and J. R. Primack, Phys. Rev. D 43, (1991) 3155.

[15] D. B. Bennett and F. R. Bouchet, Phys. Rev. Lett. 60, (1988) 257; in: The Formation and Evolution of Cosmic Strings, eds. G. W. Gibbons, S. W. Hawking and T. Vachaspati (Cambridge Univ. Press, Cambridge, 1990) p. 359.

[16] T. Vachaspati and A. Vilenkin, Phys. Rev. Lett. 67, (1991) 1057.

Figure Captions

Figure 1 : The intercommuting configuration.

Figure 2 : The “zipper”.

Figure 3 : Escape velocity dependence on the ratio of the couplings, β, for $l_{\mathrm{eff}} = 20 m_S^{-1}$ and relative angle $\alpha = 0°, 30°, 45°, 60°$.

Figure 4 : Escape velocity dependence on β for $l_{\mathrm{eff}} = 5, 20, 50, 100 m_S^{-1}$ and relative angle

a) $\alpha = 30°$

b) $\alpha = 45°$

c) $\alpha = 60°$

Figure 5 : Escape velocity dependence on the angle of approach α for $l_{\mathrm{eff}} = 5, 20, 50, 100, 200 m_S^{-1}$ and $\beta = 2$.